B.M.S COLLEGE FOR WOMEN AUTONOMOUS
BENGALURU – 560004

END SEMESTER EXAMINATION – OCTOBER 2022

M.Sc. in Mathematics – II Semester

Partial Differential Equations

Course Code: MM204T | **QP Code: 21004**
Duration: 3 Hours | **Max marks: 70**

Instructions: 1) All questions carry equal marks.
2) Answer any five full questions.

1. a) Solve the Lagrange's Partial differential equation:

$$(x^2 - y^2 - z^2)p + 2xyq = 2xz$$

b) Solve: $xu_x + (x+y)u_y = u + 1$ with $u(x,y) = x^2$ on $y = 0$.

(7+7)

2. a) Show that the solution of IVP: $u_t + uu_x + au = 0,\ x \in \mathbb{R},\ t > 0,$

$u(x,0) = bx$ for $x \in \mathbb{R}$ is $u = \frac{abx\, e^{-at}}{(a+b) - be^{-at}}$

b) Find the characteristics of the equation $pq = u$ and hence, determine the integral surface which passes through the parabola $x = 0, y^2 = u$.

(6+8)

3. a) Obtain the canonical form for hyperbolic equation from the standard second order linear partial differential equation in two variables.

b) Classify the Tricomi equation $u_{xx} + xu_{yy} = 0$ for $x \neq 0$ and hence determine the canonical form for $x > 0$.

(7+7)

4. Solve: (i) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} = \sin x \ \cos 2y$

(ii) $x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} = x^2 y^3$

(7+7)

5. a) Obtain the D'Alembert's solution of one dimensional wave equation.

   b) Show that the variable separable solution of the wave equation in spherical Coordinates gives rise to Legendre differential equation.

(7+7)

6. a) Solve the Dirichlet problem in a half plane :

   $u_{xx} + u_{yy} = 0\,;\ -\infty < x < \infty\,,\ y > 0,$ subject to $u(x,0) = f(x)$; u is bounded as $y \to \infty$, by infinite Fourier transform method.

   b) Find variable separable solution of the Laplace's equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial u}{\partial \theta} + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 u}{\partial \varphi^2} = 0$$

(7+7)

7. a) Solve the following IBVP : $u_t = K\,u_{xx}\,,\ \ 0 \le x \le 1, t \ge 0\,,$ subjected to

$$u(x,0) = f(x)\,, \qquad 0 \le x \le 1, t \ge 0$$

$$u(0,t) = 0 = u(1,t), \qquad t \ge 0$$

   by using appropriate Fourier transform technique.

   b) Solve the diffusion equation $u_t = K\,u_{xx}\,,\ \ 0 \le x \le l, t \ge 0$ subjected to the conditions $u(x,0) = f(x)\,,\ 0 \le x \le l,\ t \ge 0$

$$u(0,t) = 0 = u(l,t), \qquad t \ge 0$$

   by the Fourier decomposition method.

(7+7)

8. a) Determine the Green's function for $u_t = K\,u_{xx}\,, -\infty < x < \infty,\ t \ge 0,\ \ with$

$$u(x,0) = f(x), \qquad -\infty < x < \infty.$$

   b) Find the Green's function for the following

$$u_{tt} - 9\,u_{xx} = Q_1(x)\,, -\infty < x < \infty,\ t \ge 0, \qquad \text{subjected to}$$

$$u(x,0) = 0\ , \qquad u_t(x,0) = 0\!: -\infty < x < \infty,$$

$$u \to 0, \frac{\partial u}{\partial x} \to 0 \ \ \text{as}\ \ |x| \to \infty,\ \ t \ge 0.$$

(7+7)

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